

Least Square-Based Modelling of 0.5 HP Single-Phase Induction Motor

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ABSTRACT

A single-phase induction motor is a cost-effective device for converting electrical energy into mechanical energy, making it widely used by small and medium-sized enterprises (SMEs). Understanding motor characteristics and analyzing control system performance requires precise mathematical modelling. Nevertheless, it is difficult to derive models from physical laws, and frequently overlooks fundamental elements like usage duration and environmental conditions. This study proposes using the Least Squares method to model a 0.5 HP single-phase induction motor. With an MSE of 0.0307 and an RMSE of 0.1753, the results demonstrate that the estimated model closely resembles the real system, with only minor errors. Simulink simulations demonstrate consistent delay time and settling time values across different input variations in both open and closed-loop tests. In closed-loop conditions, rise time was nonlinear, with the slowest response occurring at 220 V and the fastest at 190 V. In open-loop conditions, rise time increased linearly with input reference. These results demonstrate that, without requiring in-depth knowledge of the physical system, the Least Squares method offers a productive and useful way to create precise mathematical models of single-phase induction motors.



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1. INTRODUCTION

An induction motor is a device that converts electrical energy into mechanical energy. Induction motors are widely used for system devices that are controlled in the industry. The use of many induction motors as an actuator device in the control system. In a control system, the application of mechanical systems has a motion dynamic. The motion dynamics of a mechanical device will display the characteristics of how the system looks. The exact information on the characteristics of the dynamic system will result in the design of the relevant control system.

The characteristics of the system can be described from the response graph simulated through the device (tools) software such as Matrix Laboratory (Matlab) [1]. It requires a mathematical model of the controlled system in using MatLab. Therefore, the importance of mathematical models is to identify and gain the characteristics of a controlled system. Then, this model also used to support the engineer in designing PID controls more easily. It is known that PID control is the most widely used control method in the industry. The process of obtaining a mathematical model of a controlled system is also called system identification.

The process of obtaining a mathematical model of a controlled mechanical system requires several methods. The most common method is the method of degradation of the laws of chemical physics of the system. The model produced by this system requires special expertise related to physical science and chemistry. In addition, some physical conditions, such as service life, measurements of inductant parameters, and capacitance, are not known with certainty and accuracy, which causes the results of the resulting model to be irrelevant for the physical system used at that time. Therefore, this study identifies mathematical modeling of induction motors using the Least Squares method. This method is a practical solution in finding mathematical models that use input and output processes in the system.

The study aims to identify the mathematical model of the single-phase induction motor based on data from an experiment using the Least Squares method. It is also aimed to evaluate the accuracy of the identification model by comparing it to actual data. Furthermore, it also to obtain a mathematical model of a high-accuracy induction motor with minimal error using the Mean Squared Error (MSE). Mathematical models are produced in the form of a transfer function which functions as the relationship between the transformation of the Input Laplace with the Transformation of the Output Laplace.

The limitation of this research is using single-phase induction motor of 1 phase, 0.5 HP, 1400 RPM. Data input-output process is in the form of voltage as input and current in output, the model obtained is a low-order model, and the method of getting a model using Least Squares. The data used in this study uses real system data, namely single-phase induction motors that are directly connected to the current sensor and PZEM voltage on the input side of the single-phase induction motor.

Research on induction motor modeling can be seen in the work of Marina Konuhova, who developed motor models with and without considering current. The study showed that the model with current consideration produced faster starting current amplitude and steady-state transition [2]. Another study by Siyu Sao focused on fault diagnosis in motor modeling. The results demonstrated that the model achieved higher accuracy and could be effectively applied to induction motors [3]. However, the models in these studies overlooked variations in physical parameters, which may change over time due to continuous operation.

A good model should accurately represent system characteristics and closely approximate the physical system. The Least Squares method has been widely employed for parameter identification [4]–[9], including applications in battery monitoring, robotic arms, and large-scale systems.

From existing studies, [1]–[9], an estimated parameter of 0.5 HP single-phase induction motors is not yet available, as only research [1] is for modeling motors, but DC motors. The data used is online data from Simulink, not real data. The Least Squares method is already used for modeling but is not yet available for the 0.5 HP single-phase induction motor. Therefore, this study proposes the modeling of a single-phase induction motor using Least Squares Estimation.

2. RESEARCH METHOD

2.1. Modelling Control System

A control system is a system designed to regulate or control certain parameters of a process or physical system in order to work as expected. In the context of the technique, the control system is widely used to regulate the speed of the motor, temperature, pressure, position, current, voltage, and so on.

The control system is divided into two main types: an open-loop control system and a closed-loop control system. The open system has no feedback mechanism, whereas a closed system has feedback to correct errors between the actual output and the desired output.

System modeling is a mathematical process for representing the dynamic behavior of a physical system in the form of a mathematical model. The goal is for the system to be analyzed, simulated, or systematically controlled.

Mathematical models of the control system usually consist of several forms, namely in the form of:

1. Differential equation (for the continuous system)
2. Different equations (for discrete systems)
3. Transfer function

4. State-space Model
5. Black Box Model

In modeling physical systems such as electric motors, models are often obtained through several ways, namely, first, theoretical approaches: based on the laws of physics (e.g., Kirchhoff's law, Newton's law). The second, with an empirical approach or system identification, based on input-output data from experiments [10], [11].

Single-phase induction motors are usually modeled using resistance, inductance, and capacitance parameters for identification based on physical laws. However, because of wear and use, these parameters might change over time. Furthermore, using physical law-based approaches necessitates precise parameter measurement and specific knowledge. Under some circumstances, a number of variables that are frequently overlooked in physical systems, such as vibration and ambient temperature, can also result in inaccurate models.

2.2. Identification System with the Least Squares Method

System identification is the process of determining a mathematical model of a system based on input-output data obtained through experiments. This process is crucial when the physical model of a system is too complex for theoretical analysis or when the system parameters are not precisely known [12]. System identification is a branch of systems engineering and is widely applied in control, prediction, and simulation of dynamic systems such as electric motors, hydraulic systems, thermal systems, and others.

A single-phase electric motor is an actuator commonly used in both industrial and household applications. This motor exhibits dynamic behavior, as its outputs (e.g., speed or current) vary over time in response to changes in its inputs (e.g., supply voltage). The behavior of such a motor can be represented as a low-order system (first- or second-order), depending on the level of complexity and the modeling objectives. Since many of its physical parameters are not precisely known, system identification offers a practical solution for modeling and analysis.

The Least Squares (LS) method is a widely used parameter estimation technique in linear system identification. Its primary objective is to minimize the sum of squared errors between the actual system output and the estimated model output [4]–[9].

In system modelling, several common representations can be expressed in both the time and frequency domains, including transfer function models, state-space models, and discrete models. A transfer function is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input, assuming all initial conditions are zero. This representation is particularly suitable for linear time-invariant (LTI) systems. The general form of the transfer function, where $G(s)$ denotes the transfer function, $Y(s)$ the output, and $U(s)$ the input, can be expressed as equation (1) follows:

$$G(s) = \frac{Y(s)}{U(s)} \quad (1)$$

Representation of the second system model is the state space model (state space), where $x(t)$ is a state matrix, A is the matrix of the state system, B is the input matrix, and C is the output matrix with the shape of the vector matrix of the following state.

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (2)$$

$$y(t) = Cx(t) + Du(t) \quad (3)$$

Representation of the Discrete model can be seen in digital or real-time systems, where discrete models are used, for example:

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) + b_1 u(k-1) + b_2 u(k-2) \quad (4)$$

where:

- $y(k)$: system output (e.g., speed or motor current) at the time
- of k $u(k)$: system input (e.g., voltage) at time k
- a_1, a_2, b_1, b_2 : the parameters of the model to be estimated
- $e(k)$: noise or interference

Mathematically, a linear model of a discrete system can be written as a linear regression model, such as equation (4) above, to form an equation connecting the output with the input. The input and output equations can be written in the form of equivalent matrix notation, which is written in equation (5) as follows.

$$Y = \varphi\theta + E \quad (5)$$

With the definition of each notation, namely:

$$Y = \begin{bmatrix} y(3) \\ y(4) \\ \vdots \\ y(N) \end{bmatrix} \quad (6)$$

$$\varphi = \begin{bmatrix} -y(2) & -y(1) & u(2) & u(1) \\ -y(3) & -y(2) & u(3) & u(2) \\ \vdots & \vdots & \vdots & \vdots \\ -y(N-1) & -y(N) & u(N-1) & u(N-2) \end{bmatrix} \quad (7)$$

$$\theta = \begin{bmatrix} a_2 \\ b_1 \\ b_2 \end{bmatrix} \quad (8)$$

where E = vector error.

The solution is obtained by θ

$$\hat{\theta} = (\varphi^T \varphi)^{-1} \varphi^T Y \quad (9)$$

Equation (9) is the Least Squares Estimation (LSE) parameter estimation.

The identification of this parameter is carried out with some process test data. After obtaining the estimation parameters, the model's accuracy is evaluated by looking for errors using Mean Squared Error (MSE). The smallest error value is found from multiple test data. The most minor error from several tests of the test data that will be used as research results.

Error values are searched with the following equation:

$$e(i) = y_i - y_p \quad (10)$$

Where $e(i)$ is the error value sought, y_i is the actual output of the observation process data, and y_p is the prediction output or the output of the model resulting from the estimate. MSE and RMSE is obtained from equations (11.a) and (11.b).

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - y_p)^2 \quad (11a)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - y_p)^2} \quad (11b)$$

where n is the amount of data used to make predictions.

R-squared is a way to assess how well a mathematical or statistical model describes the observed data. The goal is to find out whether the model we create fits with real data, or how accurate the model's prediction is compared to the real data. Thus, R Square functions to see the accuracy of the model, how representative the model is of real data, and how much error there is between prediction and observation. The R-squared can be obtained from the following equation.

$$R^2 = 1 - \frac{\sum (y_i - y_p)^2}{\sum (y_i - \bar{y})^2} \quad (12)$$

The accuracy of the model's prediction is between 0 and 1. If the value is 0, then the model does not explain the data. Furthermore, the prediction is considered perfect if the value is 1. The R-squared can be negative, which is considered a worse model than the data average.

3. RESULTS AND DISCUSSION

3.1. Transfer Function of 0.5 HP Single-phase Induction Motor Modeling

The experiment used process operation data, namely input data in the form of voltage and output data in the form of current on the motor. From the calculation of parameter estimation with a first-order approach, the following parameters in Equations (13) and (14) are obtained.

$$a_1 = -0.8014 \quad (13)$$

$$b_1 = 0.0033 \quad (14)$$

So that it can be arranged in equation (15) of the discrete model representation, namely:

$$y(k) = -0.8014 y(k-1) + 0.0033 u(k-1) \quad (15)$$

From the acquisition of the above estimation parameters, the data error level is obtained from 2 equations, namely Mean Squared Error (MSE) and Root Mean Square Error (RMSE). The results of the estimates are obtained in equations (16) and (17) below.

$$MSE = 0.0307 \quad (16)$$

$$RMSE = 0.1753 \quad (17)$$

In addition to the representation of the model in discrete equations, it can also be described in the form of Discrete Transfer Functions as follows in equation (18).

$$G_z = \frac{0.003271}{z - 0.8014} \quad (18)$$

By performing several transformation methods from Discrete Transfer Functions to Continuous Transfer Functions, equations (19) to equations (22) are obtained.

$$G_{(s)} = \frac{0.3646}{s + 22.14} \quad (19)$$

$$G_{(s)} = \frac{-0.00189s + 0.3646}{s + 22.14} \quad (20)$$

$$G_{(s)} = \frac{-0.001816s + 0.3631}{s + 22.05} \quad (21)$$

$$G_{(s)} = \frac{0.3646}{s + 22.14} \quad (22)$$

The method of transforming the discrete transfer function to a continuous transfer function from the 4 equations above, respectively, namely Zero Order Hold, First Order Hold, Bilinear/Tustin, and Pole-Zero Matching. From Equations (17) to (20) above, equations (17) and equations (20) have the same value. Therefore, the next research will be discussed using the transfer function.

The results of the graph plot are obtained with input and output graph images as shown in Figure 1 below.

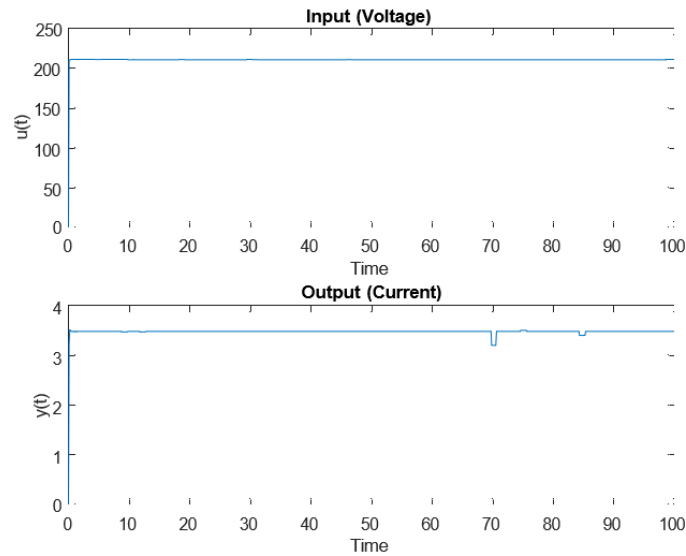


Figure 1. Graph of input and output data

From Figure 1 above, the input and output data of the induction motor system are stable. The output also fluctuates slightly at 70 and 86 seconds.

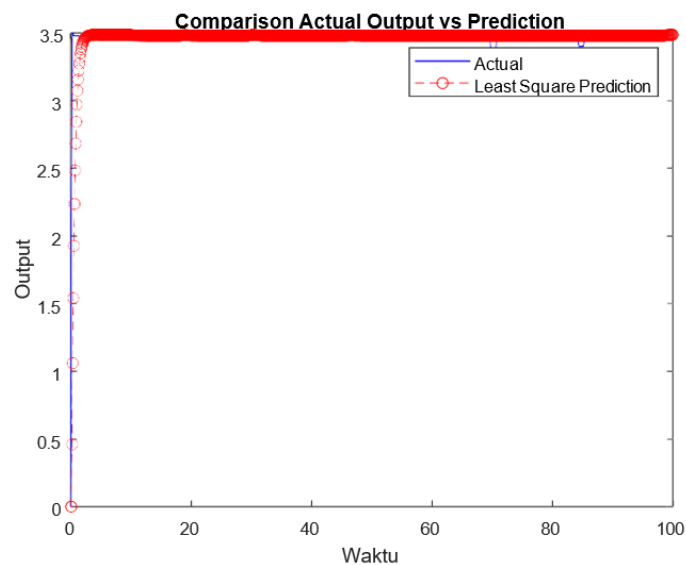


Figure 2. Comparison of Actual Output with Prediction

Figure 2 above shows that the prediction can follow the actual output with a pattern that the graph follows the desired actual output. The blue graph represents the actual output, and the red graph illustrates the predicted output.

After viewing the actual comparison graph with the prediction, the prediction error is also depicted in the form of the Prediction Output Result graph in Figure 3 below.

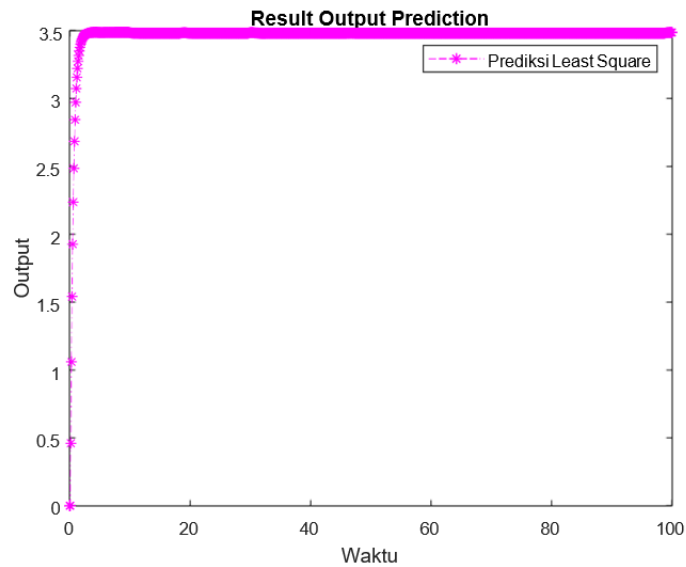


Figure 3. Prediction Output Results

The prediction output results in Figure 3 show that the response can be steady and stable at setpoint 3.5. This means that it corresponds to the actual output.

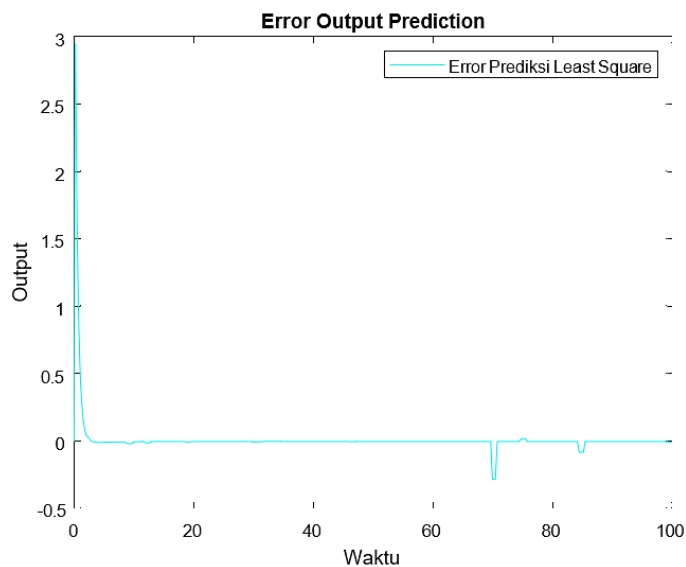


Figure 4. Prediction Output Error

From the results of the calculation of parameter estimation, the prediction output error was obtained as shown in Figure 4 above. From Figure 4, it can be seen that the prediction error is stable at Zero. This means that the parameters of the estimated results show the right value. Apart from the graph in Figure 4 above, the results of the MSE and RMSE error calculations in Equations (16) and (17) are relatively small. This points to the accuracy of the resulting model. The estimation parameter using the Least Squares method, Equations (19) to (22), is obtained, and then the Transfer Function Equation is determined to see its characteristics using Equations (19) and (22). The experiment to determine the system characteristics was carried out through open-loop and closed-loop simulations, using step input variations ranging from 180 V to 230 V. Equations (19) and (22) are changed to equation (23) below.

$$G(s) = \frac{0.016}{0.045s+1} \quad (23)$$

Equation (23) above is a single-phase Induction Motor mathematical model estimated using the Least Squares Method. An experiment was carried out using Simulink with an input step to obtain the characteristics of this single-phase Induction Motor.

3.2. Open-Loop Response Characteristics

The 0.5 HP single-phase induction motor is widely used in household industries and small to medium-sized enterprises (SMEs). The purpose of the testing is to observe and analyze the system characteristics of this motor. The experiments were carried out by applying step input voltages ranging from 180 V to 230 V. The open-loop test results are presented in Figure 5 below. An experimental image can be seen in the following Figure 5.

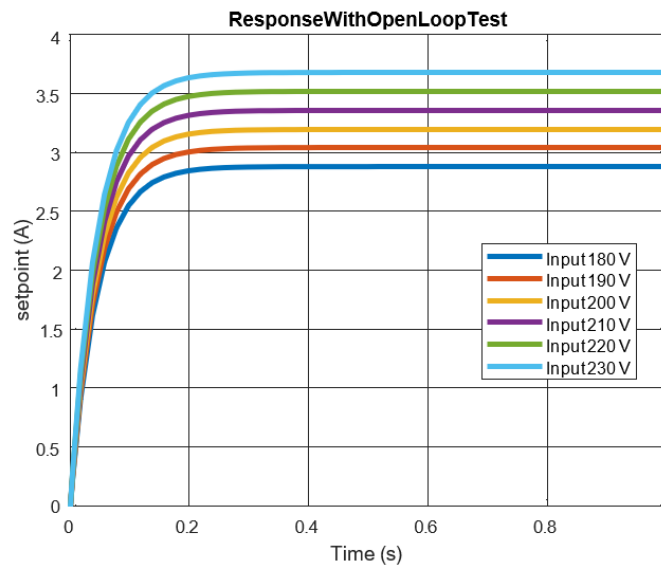


Figure 5. System Response Graph Obtained from Open-Loop Testing

The transient response analysis produced a number of important parameters, including delay time, rise time, 2% settling time, and 5% settling time, as shown in Figure 5. Table 1 presents these findings for comparison and clarity.

Table 1. Open-Loop Testing Parameters Response

Input (Voltage)	Setpoint (Ampere)	Delay Time (s)	Rise Time (s)	Settling Time 2% (s)	Settling Time 5% (s)
180	2.880	0.046	0.391	0.176	0.135
190	3.040	0.046	0.393	0.176	0.135
200	3.200	0.046	0.395	0.176	0.135
210	3.360	0.046	0.397	0.176	0.135
220	3.520	0.046	0.399	0.176	0.135
230	3.680	0.046	0.402	0.176	0.135

Table 1 provides a summary of the system response analysis's findings. All input variations result in the same delay time, 2% settling time, and 5% settling time. This consistency shows that, independent of the applied input, the single-phase induction motor's transfer function model is stable and does not significantly alter these three parameters. This stability demonstrates how well the model captures the basic dynamic properties of the motor.

On the other hand, the rise time parameter varies in response to changes in the input. In particular, the rise time slows down as the input voltage rises. At 190 V, the rise time was the fastest, and at 220 V, the slowest. In closed-loop testing, this shows a nonlinear relationship between input voltage and rise time, whereas in open-loop conditions, the rise time grows linearly with the input reference. These

results imply that while the Least Squares Estimation (LSE) approach effectively captures the fundamental motor dynamics, further improvements might be required to handle nonlinearities at higher input levels.

3.3. Closed-Loop Response Characteristics

Closed-loop testing was done on the single-phase induction motor's mathematical model, which is represented by equation (23). Figure 6 shows the obtained system response. Input variations ranging from 180 V to 230 V were used in the test.

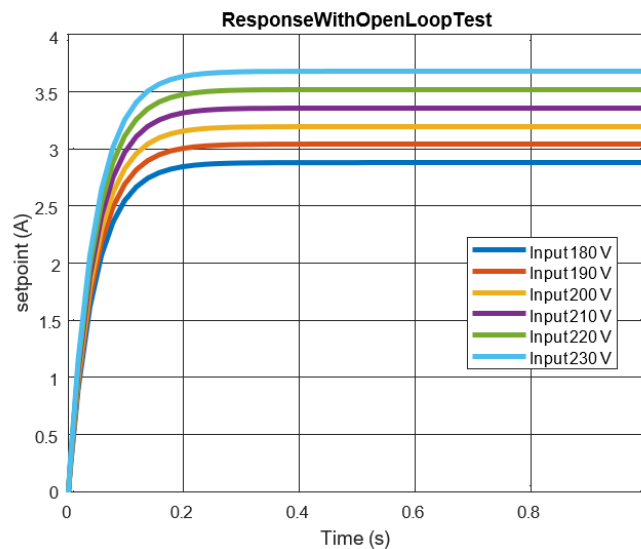


Figure 6. System Response Graph Obtained from Closed-Loop Testing

From Figure 6, the delay time, rise time, 2% settling time, and 5% settling time were obtained by observing the response graph. These parameters are summarized in Table 2, which presents the results of the closed-loop testing.

Table 2. Closed-Loop Testing Parameter Response

Input (Voltage)	Setpoint (Ampere)	Delay Time (s)	Rise Time (s)	Settling Time 2% (s)	Settling Time 5% (s)
180	2.835	0.045	0.438	0.174	0.133
190	2.992	0.045	0.376	0.174	0.133
200	3.150	0.045	0.457	0.174	0.133
210	3.307	0.045	0.384	0.174	0.133
220	3.465	0.045	0.482	0.174	0.133
230	3.622	0.045	0.391	0.174	0.133

The parameters of delay time, 2% settling time, and 5% settling time are shown in Table 2 and are unaffected by changes in the input. Regardless of whether the testing is done in a closed-loop or open-loop setting, this result shows that the single-phase induction motor remains stable with regard to these three parameters. In contrast to the closed-loop test, the open-loop test required a little more time to reach the delay time, 2% settling time, and 5% settling time.

With a value of 0.376 seconds, the fastest response was recorded for the rise time parameter at an input of 190 V, while the slowest rise time was recorded at 220 V. This suggests that rise time is dependent on input levels. The closed-loop test revealed a nonlinear relationship between input variations and rise time, in contrast to the open-loop test, which showed that rise time increased linearly with input magnitude (larger input resulted in slower rise time).

The properties of the single-phase induction motor can be examined by calculating the transfer function in equation (23), as shown in Figures 5 and 6. Equation (23) can be used to design a

proportional-integral-derivative (PID) control system in addition to determining system characteristics. Determining the parameters K_p is necessary for the design of a PID controller requires the determination of the parameters K_p , K_i dan K_d . Then. Numerous techniques, including gain analysis [13], Ziegler–Nichols tuning [14], and trial and error, can be used to determine these parameters. However, only the gain analysis method can be used for the transfer function that was derived from Equation (23).

An accurate model is produced when the 0.5 HP single-phase induction motor's parameters are estimated using the Least Squares method. The MSE and RMSE values shown in Equations (16) and (17) support this, showing a high degree of agreement between the model output and the real system response. Additionally, because the Least Squares approach only uses experimental data, it makes it easier to create a mathematical model of the single-phase induction motor.

The Least Squares method's capacity to produce models that accurately depict real-world operating conditions is another benefit. This is due to the fact that the approach makes use of input- output process data, which permits it to disregard physical parameters like capacitance, resistance, and inductance that are not measurable or change over time. Furthermore, this method yields a fairly simple mathematical model that can be approximated by models of different orders based on the design requirements. A first-order model was used in this study to approximate the motor system.

4. CONCLUSION

The estimated parameters from the experiment on using the Least Squares method to model a 0.5 HP single-phase induction motor were found to have relatively small error values and to closely resemble the actual system. The Root Mean Squared Error (RMSE) of 0.1753 and the Mean Squared Error (MSE) of 0.0307 both demonstrate this accuracy. The resulting continuous transfer function is: $G(s) = \frac{0.016}{0.045s+1}$. In both open-loop and closed-loop testing, the delay time, 2% settling time, and 5% settling time remained comparatively constant, according to the transient response analysis from the Simulink experiments with various input variations. The rise time in the open-loop setup rose linearly as the input reference values increased. The rise time, on the other hand, showed nonlinear behavior in the closed-loop configuration, with the slowest rise time occurring at 220 V and the fastest at 190 V. Without requiring in-depth knowledge of the physical system, the Least Squares approach was successful in producing a mathematical model. Future studies could incorporate the ARMAX method to further develop the Least Squares modeling approach for real-time online systems.

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