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Evaluating the Performance of High-Gain Observer (HGO) for Accurate Level Estimation in a Continuous Stirred Tank Reactor (CSTR)

Dian Mursyitah[1], Ahmad Faizal[2], Zulfatri Aini[3]
[1,2,3]UIN Sultan Syarif Kasim Riau, Jl. HR. Soebrantas KM. 15 no. 155 Simpang Baru, 28129, Pekanbaru, Indonesia

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ABSTRACT

This paper presents an estimation method for the level in a Continuous Stirred Tank Reactor (CSTR) system. Given the inherent non-linearity of the CSTR model, our research focuses on developing observers capable of effectively handling the non-linearity. The utilization of a High Gain Observer (HGO) for accurate and robust level estimation is proposed. The desired characteristics for the estimation approach include achieving minimal error, ensuring fast convergence time, and maintaining robustness in the face of noisy measurements. To comprehensively evaluate the effectiveness and robustness of the proposed observers, simulations are conducted. The results demonstrate that the HGO effectively estimates the level, closely following the trajectories of the true states within the CSTR system. Importantly, the estimation accuracy persists across various input conditions. The fast convergence of the estimation process is validated, achieving convergence within 0.1 seconds. Furthermore, robustness is confirmed as the estimation continues to closely follow true state trajectories even in the presence of noisy measurements.

Corresponding Author:

Dian Mursyitah
UIN Sultan Syarif Kasim Riau, Jl. HR. Soebrantas KM. 15 no. 155 Simpang Baru, 28129, Pekanbaru, Indonesia
Email: dmursyitah@uin-suska.ac.id

1. INTRODUCTION

CSTR are widely used in industries that require fluid mixing in their processes. However, their operation poses challenges due to their complex non-linear behavior and the associated costs involved in industrial setups [1, 2]. A common hurdle in CSTR operation is maintaining level control, especially amidst unstable flow rates, which can significantly delay achieving desired levels, especially when setpoints change. These unstable flow rates stem from variations in inflow, outflow, or system disturbances. Traditional approaches to level estimation often rely on sensors, which, over time, can become prone to inaccuracies, drift, or even failure. To maintain the desired level, feedback control systems employing level sensors and actuators are often implemented. However, this approach is hindered by the sensor's high cost, susceptibility to noise, incipient faults, and limited responsiveness due to the harsh operational environment [3, 4]. Moreover, sometimes the sensor's measurements do not contain enough information, and some signals are impossible to measure because the sensor is placed in a hard-to-reach area.

To address these challenges, advanced estimation techniques such as observer have gained prominence. The observer is the most effective-cost approach as it can reduce the reliance on a high-cost sensor. Additionally, observers can augment or even replace sensors [5]. There are several types of

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observers such as Luenberger Observer (LO), Extended Kalman Filter (EKF), High Gain Observer (HGO), and others. LO is design for linear system, providing local convergence [6]. The EKF require the modelling error to be Gaussian white noise [7]. Graton et al. have outlined criteria for choosing an observer [8]. The criteria are sensitive to noises and robust against the noise and error model. According to [9, 10], the HGO is perform well when dealing with the non-linearity with proper gain settings. Furthermore, the HGO has higher estimation accuracy and better traceability compared with the EKF as presented by [11]. The robustness of HGO has been validated in many implementation [12-14].

Given that the mathematical model of the CSTR system is non-linear, as will be presented shortly, the observers we employ are capable of accommodating this nonlinearity in the system model. Therefore, we choose to work with HGO due to its promising ability to provide accurate estimates of system states with fast convergence [7, 15, 16]. Our focus is on evaluating the performance of HGO for level estimation in CSTR and to assess the effectiveness of HGO in accurately estimating the level under varying operating conditions, different initial condition, and disturbances. The performance analysis includes two criteria: sensitivity and robustness with respect to noisy measurements, examined through simulations. The results using Luenberger Observer (LO) will be presented as comparison.

2. RESEARCH METHOD

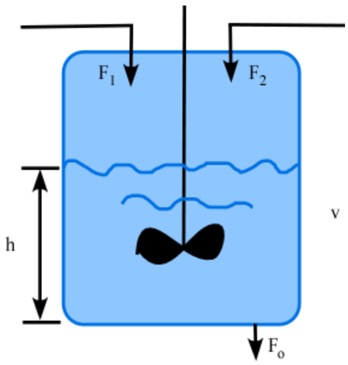
2.1. CSTR System Mathematical Model

CSTR incorporates two inputs, flow rate (F_1) and flow rate (F_2). The output flow (F_o), affects the level in the tank, assuming a perfect fluid in a tank stirred (see **Figure 1(a)**). The mathematical modeling of this system is derived from the law of tank equivalence, assuming the absence of material exiting in vapor form. This model is grounded in the principle that the volume entering the tank must equal the volume leaving it, accounting for any changes in volume within the tank.

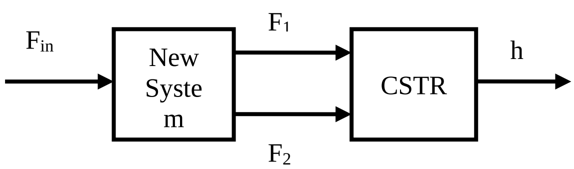
The mathematical representation of the level in the CSTR is formulated as follows [17]:

$$\frac{dh}{dt}=\frac{1}{A}(F_1+F_2)-\frac{K_c}{A}\sqrt{h} \tag{1}$$

where $h(t)$ is the level of the tank, A is the cross-section area, $F_{in}=F_1+F_2$ is the input flowrates. K_c is the a mixer constant. We can see the CSTR system in **Figure 1(a)** with the parameter of the CSTR presented in **Table 1.**



(a)



(b)

Figure 1. (a) CSTR System and decoupler (b) Decoupler design

Table 1. Parameter of CSTR System

Parameters	Units
Flow rate 1	F_1 = 0.6m^3/sec
Flow rate 1	F_2 = 0.15m^3/sec
Volume	V = 1m^3
Cross section	A = 1m^2
Mixer constant	K_c = 0.2 SI

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The level in a CSTR is characterized by a coupled system, where disturbances introduced to one input directly influence the other output and vice versa. In such a system, changes in the first input can affect the second output, and vice versa. To mitigate this coupled effect and enable the independent design of observer, a decoupler is employed. The design of a decoupler ensures that disturbances in one input do not propagate to the other as shown in **Figure 1(b).**

The design of the decoupler facilitates the transformation of equation (1) into the following form:

$$\frac{dh(t)}{dt} = \frac{1}{A}F_{in}(t) - K_c\sqrt{h}(t) \tag{2}$$

where

$$F_{in}(t) = F_1(t) + F_2(t) \tag{3}$$

with this mathematical model, we proceed to design a High Gain Observer (HGO) for estimating the level in the CSTR system.

2.2. The Design of High Gain Observer (HGO)

The HGO is used for non-linear systems and is closely related to the extended Luenberger observer [18, 19]. To enhance comprehension of HGO, let us start by creating a model of CSTR system, represented by equation (4)

$$\begin{aligned} \dot{x}(t) &= f(x,u,t) \\ y(t) &= Cx(t) \end{aligned} \tag{4}$$

where $x \in \mathbf{R}^1$ is the state system, $u \in \mathbf{R}^1$ is the input signal, and $y \in \mathbf{R}^1$ is the output of the system. The non-linear function $f(\cdot)$ related to state x, input signal u, and time *t*, are described below:

$$f(x,u,t) = \left[\frac{1}{A}\left(u - K_c\sqrt{x_1}\right)\right] \tag{5}$$

and the matrix C is presented as follows:

$$C = [1] \tag{6}$$

the HGO algorithm is presented as follows:

$$\dot{\hat{x}}(t) = A(t)\hat{x}(t) + K_H(t)\left(y(t) - C\hat{x}(t)\right) \tag{7}$$

with matrices $A(t)$ is given by:

$$A(t) = \left.\frac{\partial f(x,u,t)}{\partial x}\right|_{x=\hat{x}} = \left[\frac{K_c}{2\sqrt{x_1}}\right] \tag{8}$$

The pair of matrices *(A, C)* is observable, and observer gain is chosen as:

$$K_{HGO}(t) = \begin{bmatrix} k_{1,1}/\theta^1 & \cdots & k_{1,p-1}/\theta^1 & k_{1,p}/\theta^1 \\ \vdots & \ddots & \vdots & \vdots \\ k_{n-1,1}/\theta^{n-1} & \cdots & k_{n-1,p-1}/\theta^{n-1} \ \ddots & k_{n-1,p}/\theta^{n-1} \\ k_{n,1}/\theta^n & \cdots & k_{n,p-1}/\theta^n & k_{n,p}/\theta^n \end{bmatrix} \tag{9}$$

where $k_{1,1},\ldots,k_{n,p}$ and θ are constant with $\theta = 1$ [14]. The matrix must be chosen such that $\left(A(t) - K_H(t)C\right)$ is stable. The dynamics of the error $e(t) = x(t) - \hat{x}(t)$, can be described as

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$\dot{e}(t) = (A(t) - K_H(t)C)e(t)$. The choice of observer gain plays a crucial role in achieving accurate and fast error reduction during the estimation process. In the next section, we presented the simulation result following with the analysis.

3. RESULTS AND DISCUSSION

3.1. System behavior

The simulation was conducted using a sampling time $T_s = 0.1$ s and a simulation duration of 25s. The input signal was set to a constant value $F_{in} = 0.75m^3/s$. The simulation result is shown in **Figure 2**.

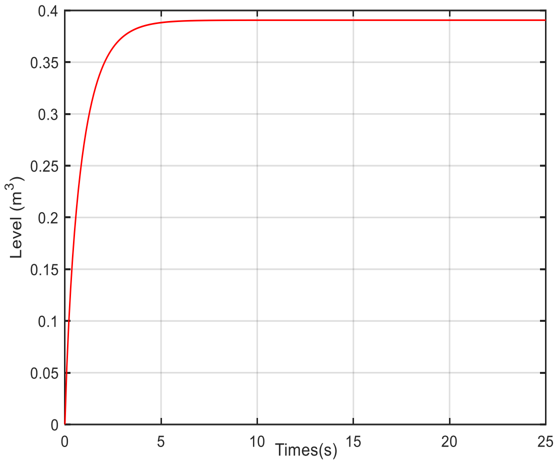


Figure 2. Output Level of CSTR System

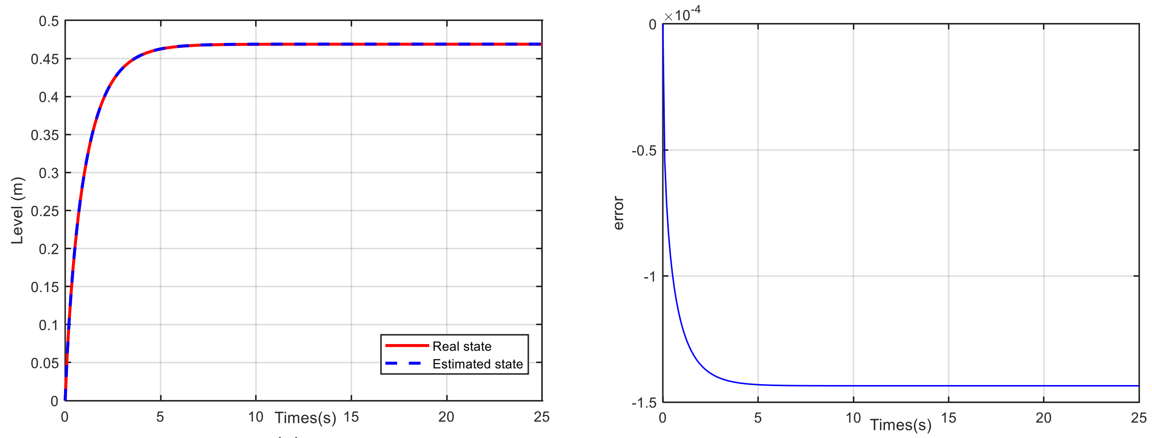
The behaviour of the CSTR system is depicted in **Figure 2**. The input signal, denote as F_{in}, is determined by adding inputs F_1 and F_2 that has been presented in (3). It seen that when the input $F_{in} = 0.75m^3/s$ is applied, the system output i.e., level stabilizes at $h = 0.39m^3/s$. The estimation results obtained using HGO will be presented in the next subsection.

3.2. States estimation performance of level in CSTR system

Based on the design in (6), the state estimation with HGO is conducted using same sampling and simulation time. The observer gain chosen for this scenario, as presented in (8) is:

$$K_{HGO} = 9.9996 \times 10^3 \tag{10}$$

The simulation result is shown in **Figure 3**.



(a) (b)

Figure 3. (a) HGO State estimation of level (b) Error estimation

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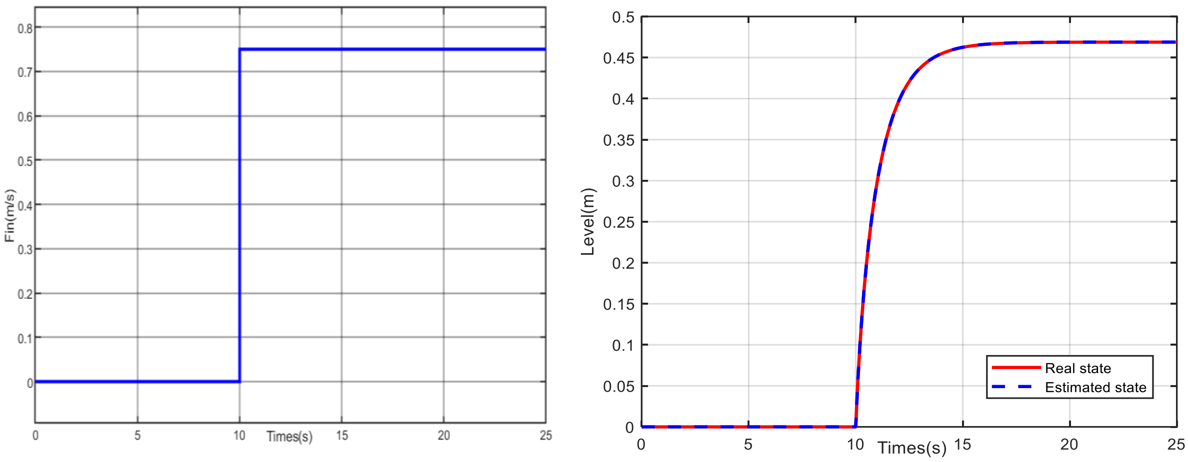
Based on **Figure 3(a),** the estimation of level given by HGO accurately reflects the true state i.e., level of the CSTR system. This alignment between the estimated and actual levels signifies the accuracy of the estimation process. Additionally, **Figure 3(b)** illustrates the error estimation, where it is evident that the error of estimation remains consistently at zero. This further validates the precision and reliability of the HGO in accurately estimating the system's state.

To conduct sensitivity analysis for the HGO, our next step involves modifying input signal to different operating points and adjusting the initial conditions for the HGO. This approach aims to evaluate the observer's sensitivity under varying operating conditions. By systematically testing the observer's performance against different setpoints, we aim to further validate its sensitivity of the observer in maintaining accurate estimations under different conditions. This analysis will help demonstrate the HGO's potential for real-time applications.

3.3. Sensitivity Analysis

1. State estimation performance of level with different input signal

To evaluate the HGO's performance under different operating points, we create an input with different operating points, transitioning from 0s to 10s and 10s to 25s, with a step shape, as illustrated in **Figure 4(a)**. The input signal undergoes a change, transitioning from 0 to 0.75 m^3/sec at 10 seconds. In the real world, we know that the flow entering the tank can vary over time and affect the level in the tank. This simulation represents real-world implementation conditions. Therefore, the simulation aims to assess the HGO's response and adaptability to different operating points in the CSTR.



(a) (b)

Figure 4. (a) Input signal with two different operating points
(b) HGO state estimation of level in CSTR system with different operating point in input signal

In **Figure 4(b)**, it is evident that the HGO maintains its ability to accurately estimate the true state and track changes in the input signal with different operating point. Notably, as depicted in **Figure 4(a)**, the input is altered at 10 seconds in accordance with the scenario provided in the input signal. This proves that even with different operating points, the HGO is still capable of providing accurate estimations. The sensitivity of HGO in handling different operation point is validated. Remarkably, these results are achieved while maintaining the same initial conditions for both the system and the HGO, which are zero. With the same initial condition of both the system and observer, we next aim to understand how sensitive the HGO is when handling different initial conditions. According to [20], the initialization of observer is an important step because it sets the starting point for the estimation process. Errors in the initial conditions can propagate through the estimation process. If the initial state estimate is far from the actual state, the observer may take a long time to converge to the correct state, leading to significant estimation errors during the transient period. In some cases, an incorrect initial condition can lead to instability in the observer. This is particularly problematic in non-linear systems. The rate at which the observer converges to the true state depends on the initial condition.

In practical applications, the true initial state may not be precisely known, necessitating assumptions or approximations. This discrepancy can cause practical issues in the performance of the

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observer. The initial condition can be start from zero and also can be start with non-zero value. Understanding the impact of initial condition variations on the observer's performance is crucial for assessing its robustness. Therefore, in this simulation, we analyse the performance of HGO to handle the different initial condition when estimating the level in CSTR system. The initial conditions are design with two different values: the initial condition for system is at 0.6m, and the initial condition for the HGO is set at 0m. This approach seeks to assess the HGO's adaptability and performance under diverse starting conditions. The state estimation results are presented in **Figure 5**.

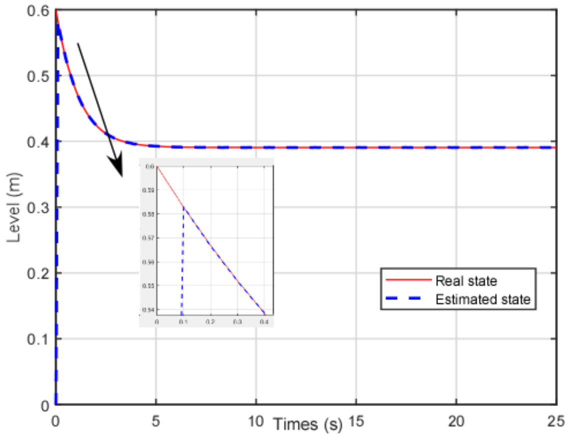

Figure 5. HGO state estimation of level in CSTR system with initial condition change

In **Figure 5**, it seen the initial condition of the system starting from 0.6m and the initial condition of HGO is from 0m. The fast convergence is validated, as shown by convergence within 0.1s. This result show that HGO is having no issues related to initial conditions, when estimating the level in the CSTR system. The estimation consistently yields favorable results, as the estimation follow trajectories the true states. To further assess the robustness of our proposed HGO, the next phase of simulation focuses on evaluating its performance with respect to measurement noise.

3.3. Robustness analysis with respect to measurement noise

In this subsection, we evaluate the performance of the HGO in handling measurement noise. The first step involves generating noise and adding it to the system. Noise is introduced into the measurements with a vector *v* as zero mean and variance unit. We describe the noise as $v: N\ (0,R)$, where R is presented as follows:

$$R = 9.5367 \times 10^{-5} \quad (11)$$

The simulation result of state estimation with noisy measurement is shown in **Figure 6**.

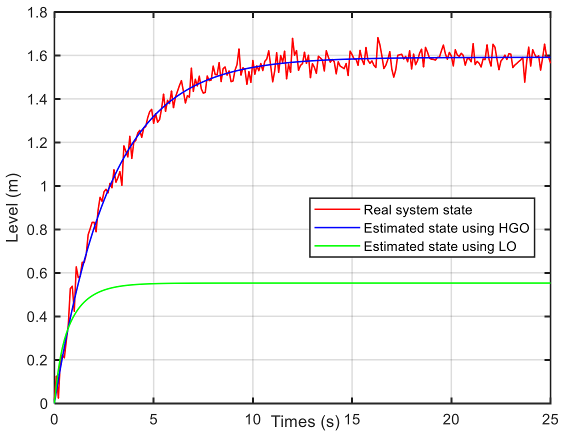

Figure 6. The HGO estimation with noisy measurements

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Figure 6 show that the real system is noisy. However, the estimation still can follow the trajectory of the true state even under the presence of noise. These results indicate that the estimation performs well under these conditions. While noisy measurements introduce challenges, HGO does not encounter difficulties in estimation. For comparison, we present the estimation from the Luenberger Observer (LO), which shows that the estimation with LO is not robust with respect to measurement noise.

4. CONCLUSION

This study evaluates the performance of the HGO for accurate level estimation in a CSTR. The proposed HGO demonstrates good performance in estimating the tank level, as evidenced by **Figure 3(a)**, which shows that the HGO accurately tracks the true state with zero estimation error. Sensitivity analysis, illustrated in **Figure 4(b),** confirms that the HGO performs effectively across different operating points, indicating its robustness in handling variations in input signals. Furthermore, as depicted in **Figure 5**, the HGO exhibits notable adaptability to different initial conditions, mitigating concerns about the system's starting conditions. The fast convergence is achieved at 0.1s. The observer also maintains robust performance in the presence of measurement noise, highlighting its effectiveness in real-world scenarios where noise is a factor as shown in **Figure 6**. These results collectively affirm that the HGO is a reliable tool for accurate level estimation in CSTR systems, validating its potential for practical application.

Future works will focus on expanding the capabilities of the HGO to include fault diagnosis in the CSTR system. This extension aims to enhance the observer's functionality by enabling it to identify and respond to potential faults or anomalies within the system, contributing to a more comprehensive and versatile control framework.

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